Name:_____

AP Calculus BC

Summer Assignment

Ms. Azorr

Room 802 B

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(It will not be checked until August- I will be in Europe! @AzorrsAbroad)

Please complete the following problems over the summer during the designated time period and bring with you the first day of school (one page a week)

Due: August 29, 2018

**** Please only use a calculator when indicated- shown by "*Calc*"**

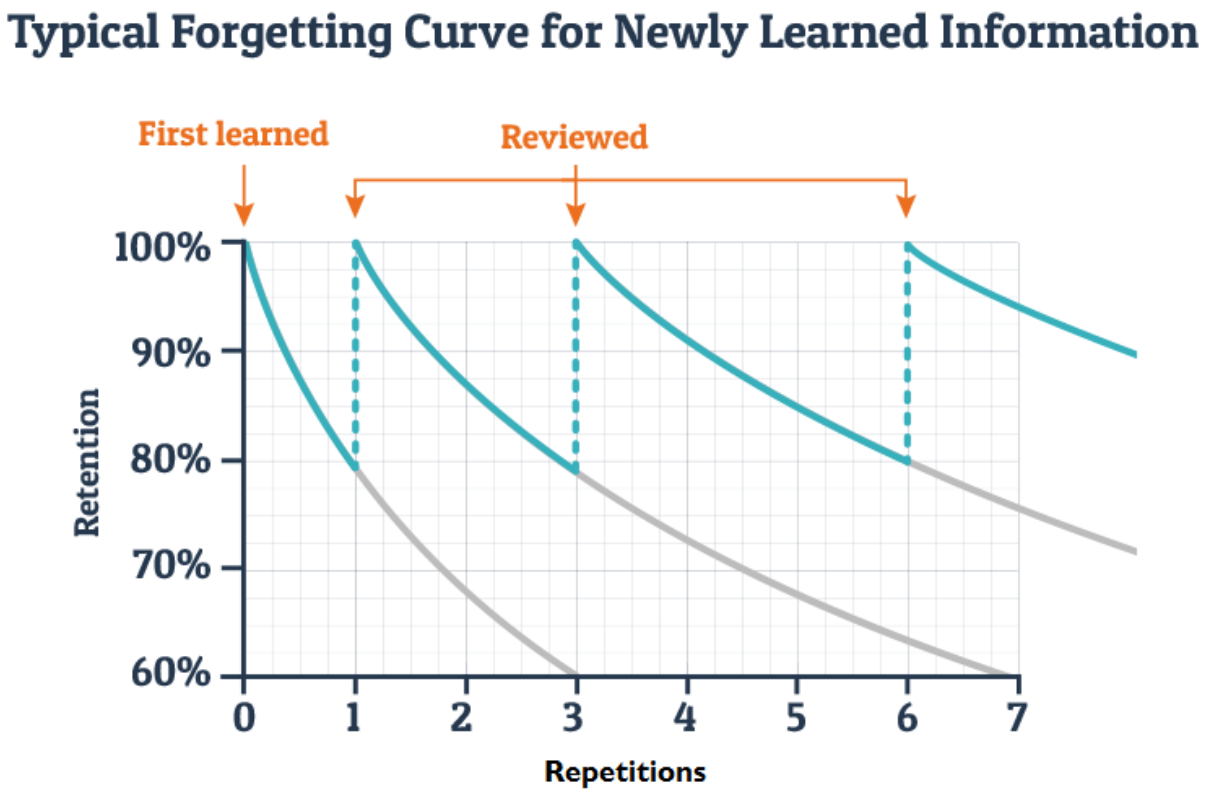
Round all calculator problems to the nearest thousandth

Why a summer assignment?

Just as all math builds on itself, AP Calculus BC is heavily based on AP Calculus AB content. You will not have as much fun next year if you have to relearn all the AB material! We will be using the same concepts that you learned this year, going further in depth and applying them in new ways. If you do just a page a week throughout the summer, you will retain more information, letting us spend more time focusing on the new material.

Have fun and good luck!

Curve of Forgetting



Studies have shown that the more often material is revisited over a period, the better the retention of the material is. If the material is revisited, your retention "jumps" up to 100% and your will retain more than the previous repetition. However, if the material is not revisited for too long, all retention lines will fall to 0% retention. DON'T FORGOT!!

Week 1: June 25 - 29th **Functions Review**

1) Simplify:

a) $\frac{x^3-9x}{x^2-7x+12}$	b) $\frac{x^2-2x-8}{x^3+x^2-2x}$

2) Rationalize the Denominator

a) $\frac{2}{\sqrt{3}}$	b) $\frac{4}{1-\sqrt{5}}$

3) Solve for x

a) $5^{x+1} = 25$	c) $\log_2 x = 3$

4) Find the difference quotient for the following $\left(\frac{f(x+h)-f(x)}{h}\right)$

a) $f(x) = 2x + 3$	c) $f(x) = x^2$

5) Evaluate. Give the exact answer and when applicable, give answers in radians $[0, 2\pi)$

a) $\cos(210°)$	b) $\sin\left(\frac{5\pi}{4}\right)$	d) $\tan\left(\frac{7\pi}{6}\right)$
e) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$	f) $\tan^{-1}(-1)$	h) $\cos^{-1}(0)$

6) Factor fully

a) $4x^3 - 8x^2 - 25x + 50$	b) $8x^3 + 27$	c) $x^4 - 1$

Week 2: July 2nd - 6th **Limits & Continuity Review**

1) Left and Right hand Limits

a) $\lim_{x\to 2^-}\left(\frac{\lvert x-2\rvert}{x-2}\right)$	b) $\lim_{x\to 2^+}\left(\frac{3}{x-2}+x\right)$	c) $\lim_{x\to 5^+}\left(\frac{5x}{5-x}\right)$

2) Limits that include Infinity

a) $\lim_{x\to\infty}\left(\frac{3x^2-4}{2-7x-x^2}\right)$	b) $\lim_{b\to-\infty}\left(\frac{\sqrt{b^2+5}}{4-3b}\right)$

3) Limits with Trigonometry / L'Hopital

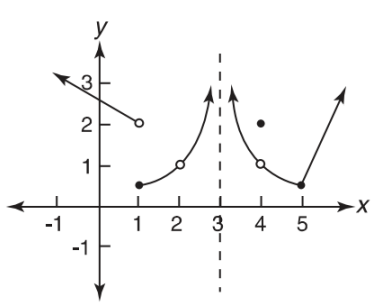
a) $\lim_{h\to 0}\left(\frac{\cos\left(\frac{\pi}{2}+h\right)}{h}\right)$	b) $\lim_{t\to 1}\left(\frac{\cos(t-1)-1}{t-1}\right)$	c) $\lim_{x\to 0}\left(\frac{x}{\sin(2x)}\right)$

4) Continuity: $f(x)=\begin{cases} x^2, & x\le 1 \\ 2x-1, & x>1\end{cases}$

a. $f(1)$	c. $\lim_{x\to 1^+} f(x)$	e. Is f(x) continuous at x=1?
b. $\lim_{x\to 1^-} f(x)$	d. $\lim_{x\to 1} f(x)$	

5) Find the value of b such that f(x) will be continuous given that $f(x)=\begin{cases}\frac{x^2-x}{x-1}, & for\ x\neq 1\\ b & ,for\ x=1\end{cases}$

6) Using the graph, for what values is f(x) discontinuous? Identify each type of discontinuity.



Week 3: July 9-13th **Derivative Rules Review**

1) Practice Those Rules!

a) $f(x) = \cos(x)\sin(3x)$, find $f'\left(\frac{\pi}{6}\right)$	b) If $y = \frac{3}{4+x^2}$, find $\frac{dy}{dx}$	c) If $f(x) = \frac{3x^2+x}{3x^2-x}$ then find $f'(x)$
d) If $f(x) = \frac{x^3}{\sqrt[3]{x}}$, find f'(x)	e) If $f(x) = 5x^{\frac{4}{3}}$ then find $f'(8)$	f) If $V = \frac{4}{3}\pi r^3$, what is $\left.\frac{dV}{dr}\right\|_{r=3}$?
g) If $y = f(x^2)$ and $f'(x) = \sqrt{5x-1}$, then find $\frac{dy}{dx}$.		i) If $y = 2e^6$, find y'

2) Implicit Differentiation

If y is differentiable function of x, then find the slope of the curve of $xy^2 - 2y + 4y^3 = 6$ at the point where y= 1.	If $x^2 - 2xy + 3y^2 = 8$, find $\frac{dy}{dx}$.

3) Derivatives of trig

If $f(x) = \sec x + \csc x$, find $f'(x)$	If $f(x) = \cos^3(x)$, find $f'(\pi)$

4) Second Derivative

If $f(x) = \cos^2 x$, find $f''(\pi)$

Week 4: July 18th-22nd **Applications of Derivatives**

1) Tangent Lines

a) If F(3)=8 and F'(3)= -4, then use the tangent line to approx. F(3.02) *Calc*	c) Approximate the slope of the tangent line of $f(x) = 4x \ln x$ at the point where x=2.1. *Calc*

2) Extrema

a) Find all x-values where $y = 3xe^{2x}$ has a relative extrema.	b) Find the area of the largest rectangle that has two vertices on the x-axis and two vertices on the curve $y = 9 - x^2$.

3) Related Rates

a) A 26 ft ladder leans against a building so that its foot moves away from the building at the rate of 3 feet per second. When the foot of the ladder is 10 feet from the building, the top is moving down at a rate of what?	b) Sand is falling into a conical pile at the rate of 10 m^3 /s such that the height of the pile is always half the diameter of the base of the pile. Find the rate at which the height is changing when the pile is 5m high. ($V = \frac{1}{3}\pi r^2 h$)

4) Velocity and Acceleration

a) A particle moves along a horizontal path so its velocity at any time t > 0 is given by $v(t) = t\ln(t)$ ft/s. Find its acceleration. (Provide units)	b) The acceleration of a particle moving along a straight line is given by $a = 6t$. If, when t=0, its velocity, v, is 1 and its position, s, is 3, then find the general equation for s.

5) Concavity

On what interval is the graph of $\frac{1-x}{x-3}$ concave upward?	Water is poured at a constant rate into a conical reservoir (point as the bottom). If the depth of the water, h, is graphed as a function of time, then the graph is (pick one): *Decreasing, Constant, Increasing, Linear, Concave Up, Concave Down*

Week 5: July 25-29th **Integral Review**

1) Antiderivatives

a) Find the antiderivative of $\frac{3}{x^2}$	b) If $\frac{dy}{dx} = \pi y$, what could y be? (separable differential equation, or direct variation)

2) U-Substitution/Definite Integrals

$\int_1^{e^3} \frac{\ln x}{x}\,dx$	$\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}}\,dx$	$\int_0^3 \sqrt[4]{2t+1}\,dt$

3) Indefinite Integrals (recall the **formula**, you shouldn't be doing too much computing!)

$\int \frac{5}{\sqrt{9-4x^2}}\,dx$	$\int 3^{2x}\,dx$

4) Trigonometry Integrals (HINT: You may have to look up some trig ID's!)

$\int \sin x \cos^2 x\,dx$	$\int_0^{\pi} \cos\left(\frac{x}{2}\right)dx$	$\int \sin^2(x)\,dx$ (power reducing)

5) FTC's FTW!!

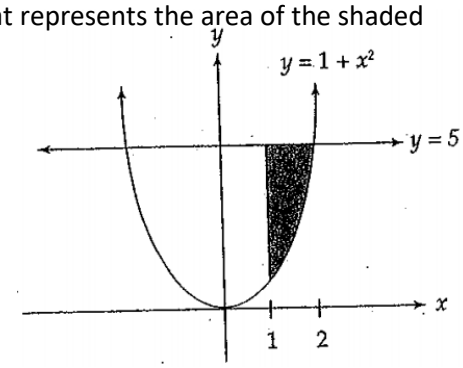
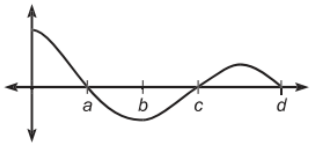
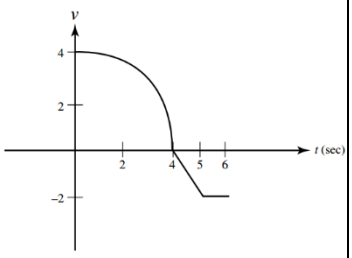
a) $\frac{d}{dz}\left[\int_0^{2x} e^{4z^2}\,dz\right]$	b) If $F(x) = \int_0^{2x} \frac{1}{1-t^3}\,dt$, then find F'(x).

Week 6: July 30st-August 3rd **Applications of Integrals**

1) Average Value

a) Find the average value of $y = e^{3x}$ over the interval from x=0 to x=4.	b) Find the average (mean) value of $\tan x$ on the interval from x=0 to $x = \frac{\pi}{3}$

2) Accumulation/ Pictorial Representation

a) *Calc* What is leaking from a tank at the rate of $R(t) = 5\arctan\left(\frac{t}{5}\right)$ gallons per hour, where t is the number of hours since the leak began. How many gallons will leak out during the first day?	Write an integral that represents the area of the shaded region 
 The graph of f is shown above. If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a relative minimum? **(A)** a **(B)** b **(C)** c **(D)** d **(E)** It cannot be determined from the information given.	The graph consists of a quarter circle and 2 line segments and represents the velocity of an object for 6s interval Find the object's avg speed over the 6s interval. Find the object's acceleration at t=4.5s. 

4) Riemann Sum

Which of the following is equivalent to $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(1+\frac{2i}{n}\right)^2+1\right]\left(\frac{2}{n}\right)$? **A.** $\int_2^4 (x^2+1)\,dx$ **B.** $\int_1^3 x^2\,dx$ **C.** $\int_1^2 (x^2+1)\,dx$ **D.** $\int_1^3 (x^2-1)\,dx$ **E.** $\int_1^3 (x^2+1)\,dx$	If L(n), R(n), M(n), and T(n) denote left, right, midpoint, and trapezoidal sums with n subdivisions for $\int_{-3}^{3}(-x^2+9)dx$, compute the following: a) L(2) b) M(2) c) T(3) d) M(4) e) R(6)

Week 7: August 6-10th **Area and Solids in the Coordinate Plane**

1) Area of Bounded Region

a) Find the total area of the region bounded by $y = x\sqrt{1-x^2}$ and the x-axis.	b) Find the area of the region bounded by $y = e^{2x}$, the x-axis, the y-axis, and the line x = 2.
c) *Calc* Find the area of the first quadrant region bounded by $y = x^2$, $y = \cos(x)$, and the y-axis.	d) *Calc* Approximate the area of the region completely bounded by $y = \ln(x) + 2$ and $y = 2x$.

2) Volume of base with known Cross-Section

a) The base of solid is the region bounded by the parabola $y^2 = 4x$ and the line x=2. Find the volume of the solid if the cross section perpendicular to the x-axis a square.	*Calc* The base the volume of a solid is the region bounded by the curve $y = 2 + \sin(x)$, the x-axis, x=0, and $x = \frac{3\pi}{2}$. Find the volume of the solids whose cross sections are semi-circles perpendicular to the x-axis.

3) Revolution about an axis

a) Set up, but do not solve the integral that represents the volume of one arch of the sine curve about the x-axis.	b) *Calc* Find the volume of the solid generated when the region bounded by the y-axis, $y = e^x$, and y=2 is rotated about the x-axis.

Week 8: August 13-17th **FRQ Practice without Calculator**

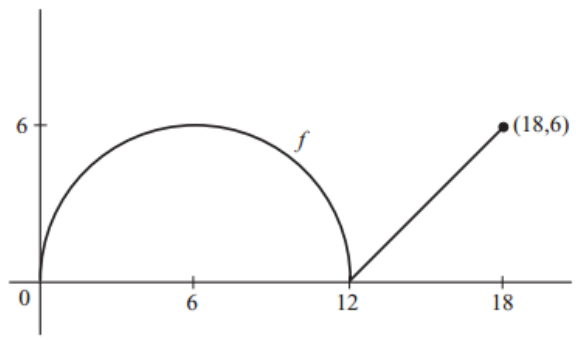
1. Let C represent the curve determined by $f(x) = \frac{6}{\sqrt[3]{2x+5}}$, for $-2 \leq x \leq 11$.
   a. Let R represent the region between C and the x-axis. Find the area of R.

   b. Set up, do not solve, and equation to find the value of k such that the line x=k divides R into 2 regions of equal area.

   c. Set up and integral for the volume of the solid generated when R is rotated around the x-axis.

2. The graph of f consists of the semicircle and line segment shown in the figure. Define the area function for $0 \leq x \leq 18$

$$A(x) = \int_0^x f(t)dt$$



   a. Find A(6) and A(18)

   b. What is the average value of f on the interval?

   c. Write the equation of the line tangent to the graph A at x = 6.

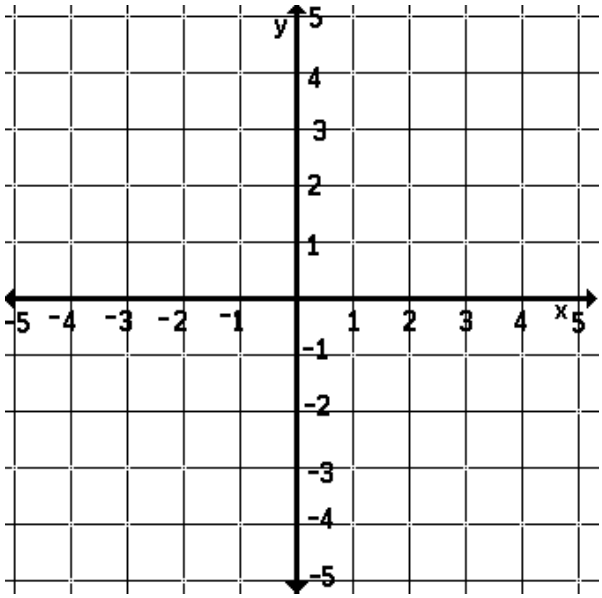
   d. Use this line to estimate the area between f and the x-axis on (0,7].

   e. Give the coordinates of any points of infection on the graph of A. Justify your answer.

Week 9: August 20th-24th **FRQ Practice with Calculator**

1. Let h be a function that is even and continuous on the closed interval [-4,4]. The function h and its derivatives have the properties indicated in the table. Use this information to sketch a possible graph of h on [-4, 4].

x	$h(x)$	$h'(x)$	$h''(x)$
0	$-$	0	$+$
$0 < x < 1$	$-$	$+$	$+$
1	0	$+$	0
$1 < x < 2$	$+$	$+$	$-$
2	$+$	0	0
$2 < x < 3$	$+$	$+$	$+$
3	$+$	undefined	undefined
$3 < x < 4$	$+$	$-$	$-$



2. An object in motion along the x-axis has velocity $v(t) = (t + e^t)\sin(t^2)$ for $1 \le t \le 3$.
   a. Sketch the graph of the velocity as a function of time in the window [1,3]x[-15,20].

   b. When is the object moving to the left?

   c. Give one value of t from the interval in part (b) at which the speed of the object is increasing.

   d. At t = 1 this object's position was x = 10. What is the object when t = 3?

Week 10: August 27-29th **College Preparation**

If you have not, check your AP Calculus AB score:

1. Log onto apscore.org (you may have to create an account).
2. You will need your AP number.
3. Locate your score: _____

Select your top 3 colleges you would like to attend next fall. Go to their website for AP credit and fill out the following table:

College	AP Test	Score	Class Equivalent
	AP Calculus AB	3	
		4	
		5	
	AP Calculus BC	3	
		4	
		5	
	AP Calculus AB	3	
		4	
		5	
	AP Calculus BC	3	
		4	
		5	
	AP Calculus AB	3	
		4	
		5	
	AP Calculus BC	3	
		4	
		5	

What is your desired score on the AP Calculus BC Exam? _____